Math 10B with Professor Stankova
Worksheet, Discussion #13; Thursday, 3/7/2019
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1 Discrete Probability Distributions

1.1 Concepts

1.

Distribution	PMF	Example
Uniform	If $\#R(X) = n$, then $f(x) = \frac{1}{n}$ for all $x \in R(X)$.	Dice roll, $f(1) = f(2) = \cdots = f(6) = \frac{1}{6}$.
Bernoulli Trial	$f(0) = 1 - p$, $f(1) = p$	Flipping a biased coin
Binomial	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$	p is probability of success. Repeat n Bernoulli trials. Number of 6's rolled when rolling 10 die is $f(k) = \binom{10}{k}(1/6)^k(5/6)^{10-k}$.
Geometric	$f(k) = (1-p)^k p$	k failures and then a success.
Hyper-Geometric	$f(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	Counting the number of red balls I pick out of n balls drawn if there are m red balls out of N balls total.
Poisson	$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$	Count the number of babies born today if on average there are 3 babies born a day.

1.2 Examples

2. I am picking cards out of a deck. What is the probability that I pull out 1 heart out of 5 cards if I pull with replacement? If I pull 5 cards at once?

3. What is the probability that first heart is the third card I draw (with replacement)?

1.3 Problems

4. True False We cannot talk about Bernoulli trials for rolling a 5 because there are 6 outputs and we need 2 for a Bernoulli trial.

5. True False When flipping a fair ($p = 1/2$) coin, the probability of flipping 100 heads on 200 flips is 50%.

6. True False The geometric distribution, like the hyper-geometric distribution, assumes that the trials are dependent (without replacement).

7. In a class of 50 males and 80 females, I give out 3 awards randomly. What is the probability that females will win all 3 awards if the awards must go to different people? What about if the same person can win all three awards?

8. Suppose I am baking souffles and each souffle has a 10% chance of not rising. What is the probability that when baking 50 souffles for class, at most 2 of them will not rise? Suppose that I keep baking them until a souffle doesn't rise. Let X be the number of souffles I bake before one doesn't rise. What is $P(X \geq 20)$?

9. At Berkeley, there is an equal number of people aged $18, 19, \ldots, 27$. I cold call someone at random and ask for their age. What is the PMF for their age? Suppose that undergraduates are aged 18 though 21 inclusive. What is the probability that I have to call 10 people until I call an undergraduate (the undergraduate is the 10th person I call)? What is the probability that I call 4 undergraduates out of 10 people I call (if I can call someone more than once)?

10. (Challenge) For a lottery, 6 distinct numbers are drawn out of 60 and to win, you need to match all 6 numbers. What is the probability that I win? If I buy 100 different tickets, what is the probability that I win?

1.4 Examples

11. On average, there are 20 rainy days in Berkeley per year. What is the probability that this year, there are 30? What is the probability that over 5 years it will rain 100 times?

12. The probability of seeing a shiny Pokemon is approximately 1 in $10000 = 10^4$. What is the probability that I don't see any in my playthrough if I see 10^5 Pokemon total? (calculate both exactly and an approximation)

1.5 Problems

13. True False We can use the Poisson distribution to show that $\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} = 1$.

14. When a cell undergoes mitosis, the number of mutations that occurs is Poisson distributed and an average of 11 mutations occur. What is the probability that no more than 1 mutation occurs when a cell divides?

15. The number of chocolate chips in a cookie is Poisson distributed with an average of 15 chocolate chips. What is the probability that you pick up a cookie with only 10 chocolate chips in it?

16. The number of errors on a page is Poisson distributed with approximately 1 error per 100 pages of a book. What is the probability that a novel of 300 pages contains no errors?

17. (Challenge) Approximately 4 people are born every second. What is the probability that in a minute, there are 240 people born?